Design and Operation of Aircraft to Minimize Their Sonic Boom

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Means of reducing or eliminating the sonic boom through aerodynamic design or aircraft operation are discussed. These include designing aircraft to minimize or eliminate certain features of the overpressure signature, operating aircraft at slightly supersonic speeds so that the sonic boom does not reach the ground, and seeking reductions through the high-altitude, high-speed flight conditions of hypersonic transports. A simple computer program has been developed that determines the area development of the equivalent body of revolution required to minimize various sonic boom signature parameters.

I. Introduction

POOR range and payload characteristics, sideline noise, and, as a result of the sonic boom, restriction to subsonic operation over land areas have constrained supersonic transports from becoming a significant part of the world's transportation system. The sonic boom of such aircraft is one of the simplest of noises; it also seems to be one of the most inevitable and one of the more noxious ones.

Perhaps the next round of SST designs will result in aircraft with greatly improved range and payload characteristics; some of the proposed designs certainly hold that promise. If so, then we will have to know whether or not design changes can and should be made that will reduce the sonic boom to acceptable levels. On the other hand, such aircraft may be most attractive when operated at only slightly supersonic speeds where, thanks to the tropospheric sound speed gradient, the sonic boom may be avoided at the ground. Or it may be that none of these possibilities will prove attractive and our attention will turn to hypersonic transports. If so, we should have some idea whether or not such aircraft will also be constrained from supersonic operation over land areas. For if they are, they too may prove uneconomic and we should use caution in investing in their development.

An earlier paper[1] gave a brief history of the subject of sonic boom minimization and detailed various means and procedures for reducing the sonic boom of supersonic aircraft. To a large extent this paper draws on the information contained in the earlier paper, and it should be considered a sequel to it. While we review some of the general considerations of sonic boom minimization here, our review will be brief. The reader is referred to the earlier paper for more substantive details. Other results pertinent to this paper and upon which we expand here are to be found in Ref. (2).

One of the primary difficulties with this field is knowing what is to be reduced or minimized in order to make the sonic boom more acceptable. As it is experienced outdoors, the most annoying feature of the sonic boom is the shock wave that gives rise to the more descriptive appellation used in Europe, "sonic bang." When the sonic boom is experienced indoors, however, one of the most significant parameters is undoubtedly the impulse of the signature, that is, the integral of the pressure with time over that period of time during which the pressure is positive. It is possible to design aircraft whose pressure signature is not preceded by a shock wave; in this case the important parameters are the time it takes the pressure to rise to its maximum value, that maximum value of the pressure, and the impulse. The maximum pressure level in the sonic boom signal is usually referred to as the overpressure. We can also envision pressure signatures that are preceded by a shock wave and then take a finite time to rise to the maximum pressure. Thus there are a number of quantities that will determine the impact of the sonic boom. These include the shock strength, the overpressure, the impulse and the rise time. Determining the weights to assign to these various signature parameters is at best a difficult and perhaps even an impossible task. Some progress has been made toward finding a composite quantity that describes the effects of sonic booms on simplified structures.[3,4] The problem of "acceptability" is less well understood.[5] As aerodynamicists we must be prepared to minimize some weighted combination of the significant parameters. However, we know so little about the weights to assign to the various parameters that such a procedure would be an academic exercise. Consequently, at this stage of our knowledge of the "impact" of the sonic boom, we proceed simply by minimizing any one of these signature parameters with no constraint on the others.

There are various ways to attempt to reduce the sonic boom. To begin with, it is important to note that any improvement in the traditional parameters that govern the efficiency of the aircraft will usually result in a reduction of the sonic boom overpressure and impulse. Improvements in the lift-drag ratio, the thrust-weight ratio, the specific fuel consumption and the structural weight usually mean sonic boom reductions. While it is often thought that an increase in the altitude of flight will reduce the sonic boom we shall see that this is not necessarily the case. Aside from these usual measures of the efficiency of the aircraft, we can try to minimize any one of the various pressure signature parameters by careful aerodynamic design of the aircraft. It is this procedure that is the main subject of this paper. Only if we have very limited success with this technique does it make sense to examine more exotic approaches. A number of such exotic schemes have been proposed over the last five years; some of these are rational, others violate the basic laws of physics. To a large extent these embody the addition or removal of heat from the flow. Linear theory tells us, however, that sources of heat and mass affect the aircraft's equivalent body of revolution in a simple additive way. This implies that for mass or heat addition (or removal) to be effective in changing the aircraft's equivalent body of revolution the mass or enthalpy flux added to the flow must be comparable to the aircraft's lift coefficient times the freestream mass or enthalpy flux through an area equal to the

Presented as Paper 73-241 at the AIAA 11th Aerospace Sciences Meeting, Washington, D.C., January 10–12, 1974; submitted March 26, 1973; revision received May 8, 1974. This research was supported by NASA under Grant NGR-33-010-203.

Index categories: Aircraft Aerodynamics (Including Component Aerodynamics); Aircraft Noise, Aerodynamics (Including Sonic Boom; Supersonic and Hypersonic Flow.

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aircraft's wing area. This simple observation presumably rules out the practical application of such concepts. More careful analyses vindicate this presumption.[6-8]

II. Aerodynamic Minimization

There are important tradeoffs between the aerodynamic minimization of sonic boom and aircraft performance. The most important question to address at this time is whether aerodynamic minimization is likely to achieve significant enough reductions in sonic boom levels to allow the overland operation of supersonic transports at supersonic speeds. Consequently, we ask to what extent we can reduce the sonic boom by aerodynamic means? Specifically, we ask how do we minimize one of the signature parameters without any constraint imposed on the others or on the aircraft's performance? We assume that the aircraft's volume may be achieved without penalty and that the aircraft is simply characterized by its weight and effective length. For the purposes of standardization we use a ground reflection factor of 2 and advocate that others do likewise. For simplicity we assume that the atmosphere is isothermal as this makes it easy to compare results with those of other investigators; results for an isothermal atmosphere differ but slightly from those for a real atmosphere at altitudes up to 200,000 ft.[9] We then use the supersonic area rule and pose the appropriate question in terms of the aircraft's equivalent body of revolution for the vertical azimuthal plane; at this juncture we have not concerned ourselves with the minimization of any integrated quantities over the width of the sonic boom corridor. We ask, then, how we shape the equivalent body of revolution for the vertical plane to minimize a given signature parameter below the aircraft?

Supersonic Area Rule

Perhaps we should digress at this stage to mention that the concept of the area rule[10,11] has come into question on several occasions in the past. The most recent and notable questioning of this result is due to Oswatitsch and Sun.[12] They argue that the plane expansion wave emanating from the trailing edge of a delta wing with supersonic leading edges will erode the front conical shock of such a wing to the degree that, at a distance of e^2 times the vertical distance from the wing to the point where the first Mach wave from the expansion meets the shock, the shock will vanish. They note that "the general nonequivalence of a wing to a body of revolution in this respect should evoke some re-thinking about sonic boom prediction and alleviation."[12] We have examined their analysis of this problem and find it in error on two counts. We will address the correct calculation in a later paper. First, and most important, we note that the expression [(Eq. (40)] they give for the characteristics (and thus the flow properties) in the expansion fan from the trailing edge of the delta wing is only valid until the first characteristic of the fan intersects the shock. Thereafter flow changes across the characteristics of the fan must be referenced to the actual pressure behind the three-dimensional shock and not to the pressure that existed ahead of the fan before it met the shock. If this expression is replaced by the proper one, then one finds that the front shock will decay as the inverse first power of the distance below the aircraft. The full calculation supporting these contentions has been carried out by M. Cramer[13] and will be reported in a forthcoming publication. Secondly, in the linear approximation the radial distance between the wing tip cones and the trailing edge expansion in the plane of symmetry is simply $b^2/8r$, where b is the wing span and r the radial distance below the wing; this distance soon becomes negligible compared to the wing span. When the wing tip effects interact with the shock, its rate of decay begins a transition from the inverse first power to the quasi-axisymmetric inverse three-quarters power of the distance. There have been numerous experimental vindications of the supersonic area rule and a theoretical search[14] for cumulative second-order effects in the azimuthal direction indicates that there are none if the disturbances are small. There are, of course, second-order corrections that become important at higher Mach numbers, but they do not constitute a violation of the supersonic area rule.

Aerodynamic Design

The deduction of the shape of the body of revolution that minimizes a given signature parameter below the aircraft is equivalent to the specification of the Whitham F function for that azimuthal plane. With a few well-known facts in hand, intuition may be used to "guess" the proper minimum. The mathematical formulation of this problem is outlined in the appendix. That this is a minimum can then be proved, either by considering infinitesimal variations of the F function or by using bang-bang control theory techniques introduced by Hayes and Weiskopf[15] for the shock-free (bangless boom) case. The important facts are that the signal from the aircraft is essentially acoustic in nature but that it is of finite amplitude. An acoustic signal obeys the simple rule of geometric acoustics

$$p_1{}^2 S/\rho a = \text{const}$$

where p_1 is the perturbed pressure from linear theory, S is the ray tube area, and ρa, the product of the density and the sound speed, is the acoustic impedance. Thus, for an isothermal atmosphere, in the absence of nonlinear effects, the signal would decay as

$$p_1 = (Pz_*/P_*z)^{1/2}p_{1*}$$

where the asterisk subscript refers to some reference conditions near the aircraft and P is the ambient pressure. Because the signal is of finite amplitude compressions steepen. The amount they steepen at any phase of the signal is proportional to the amplitude there. Thus there is a nonlinear advance of some parts of the signal relative to other parts. In a homogeneous atmosphere this advance proceeds indefinitely; in an isothermal atmosphere, however, the increasing acoustic impedence below the aircraft means that the advance approaches a finite limit. As Hayes[16,17] noted, the asymptotic advance achieved in an isothermal atmosphere below an aircraft is the advance that occurs in a homogeneous atmosphere in a distance of $\pi/2$ isothermal scale heights. Because of the nonlinear advance then, shock waves will appear. Once shock waves appear the decay of the pressure signal is enhanced by a factor that is asymptotically proportional to the inverse one-half power of this advance. It is these facts that allow one to deduce the appropriate prescription for the Whitham F function to minimize given signature parameters. Because the pressure perturbation that must be induced by the aircraft to support itself is independent of altitude, while that induced by a given flow deflection angle due to the aircraft's volume is proportional to the ambient pressure, the lift contribution to the sonic boom decays less rapidly than that due to volume by a factor of exp $(h/2H)$, where h is the aircraft's altitude and H an (isothermal) atmospheric scale height.

The general rules for this procedure are shown in Fig. 1. We represent the aircraft as a black box moving at a supersonic Mach number in a stratified atmosphere. At some large distance from this aircraft a pressure disturbance is sensed. We can think of that disturbance as being caused by a body of revolution with some effective base area equal to $(M^2-1)^{1/2}$ times the aircraft's weight divided by twice the dynamic pressure plus any stream tube area changes caused by the engines. This ultimate base area is achieved in some effective length l_{eff}. Note that the effective length of an aircraft ranges from its true

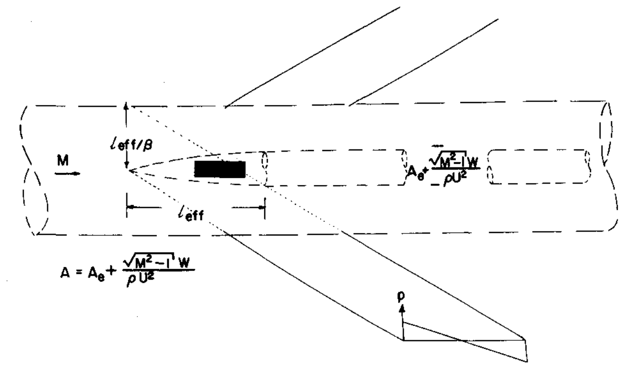

Fig. 1 General representation of flow field.

length to l_{eff}/β depending on the ratio of its horizontal to vertical extent. Frequently we will identify this length with the length of the aircraft l.

Aircraft Area Development

For a given ratio of front-to-rear shock strength, we consider the minimization of the following quantities: the impulse; the overpressure; the pressure rise through the shocks. Signatures with these quantities minimized are depicted in Fig. 2. In the first signature, p_J is the pressure rise through the shock when the impulse is minimized; the subscript "J" refers to L.B. Jones, who first minimized this quantity for the case of flight in a homogeneous atmosphere. The second signature is one in which the maximum pressure in the signature, the overpressure, has been minimized, and the third, a signature in which the pressure rise through the shock has been minimized

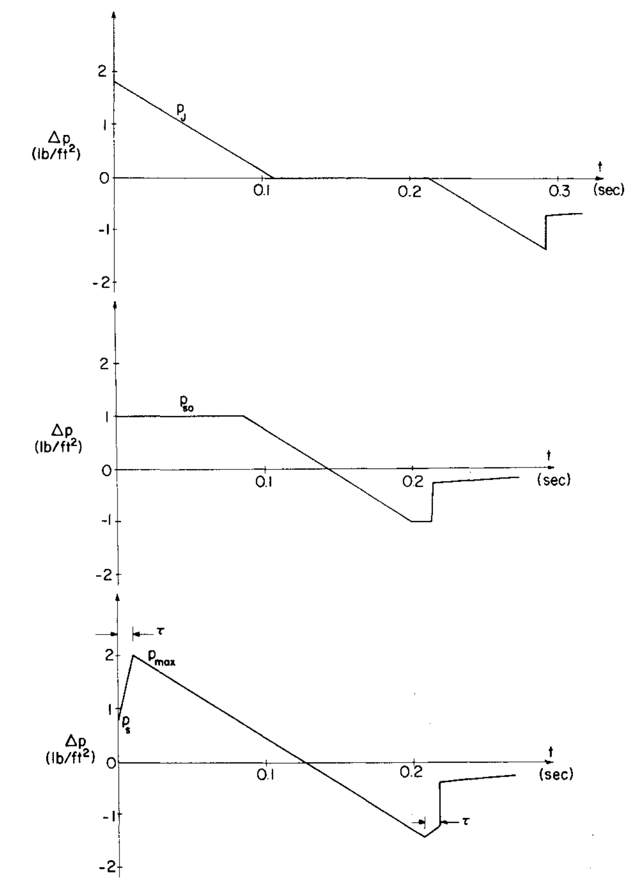

Fig. 2 Overpressure signatures considered. Except in the minimum impulse case the front and rear shocks are taken to be the same strength. The numbers given correspond to the results for M = 2.7, W = 600,000 lb, l = 300 ft, h = 60,000 ft, τ = 0.01 sec.

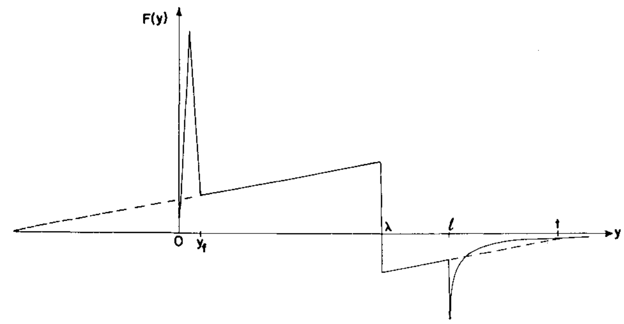

Fig. 3 Whitham F function for minimizing various signature parameters.

under the assumption that the pressure rise after the shock occurs in a time τ small compared to the period at the positive phase of the signal. Most of our results actually apply to the limiting case $\tau \to 0$. The arguments that lead to the deduction of the aircraft area development to minimize these quantities are given in various papers by the authors.[1,2,18-20] We summarize them very briefly here. (See also the papers by Jones,[21-23] Hayes and Weiskopf,[15] Ferri,[24] and Petty.[25])

First, recall that far from a body of revolution the pressure field is proportional to the Abel transform of the second derivative of the cross-sectional area of the body[11] by

$$F(y) = \frac{1}{2\pi} \int_0^y \frac{A''(x)}{(y-x)^{1/2}}\, dx$$

This function is often called the Whitham F function. A simple inversion of this formula leads to a relationship between the weight of the aircraft, which is proportional to the base area of the equivalent body of revolution, and the F function:

$$A(l) = A_e(l) + \frac{\beta W}{\rho U^2} = 4 \int_0^t F(t)\,(l-t)^{1/2} dt$$

A further quadrature relates the volume and center of pressure location to another weighted integral of the F function. Discontinuities in the first derivative of the cross-sectional area can be most easily handled by considering the function to be a generalized one. Once the F function is prescribed over the length of the body, the equation[1]

$$F(t) = -\frac{1}{\pi(t-l)^{1/2}} \int_0^t \frac{t(l-x)^{1/2}}{t-x} F(x) dx$$

determines it for all $t > l$.

We note from the equation for $A(l)$ that we gain the most by having F as large as possible as soon as possible. That is, if we invert the problem and ask for a given overpressure or shock pressure rise, how can we maximize the weight of the aircraft, then we see that we should make F as large as possible as soon as possible. However, if shock waves are not to occur then there is a limit to the rate at which F can grow if the signal is not to steepen into shock waves. This leads to a restriction on the maximum value of F'. However, if shocks are to occur, then they decay more rapidly if they are introduced immediately (that is, at the aircraft). This we do by beginning the F function with a delta function multiplied by an undetermined coefficient $\mathcal{A}$. Thus we are led to consider F functions of the form shown in Fig. 3 with shocks introduced by the area balances indicated by the dashed lines,[18] and the F function will have the form

$$F(y) = \begin{cases} \mathcal{A}\delta(y) + \mathcal{B}y + \mathcal{C}, & 0 \le y \le \lambda \\ \mathcal{B}y - \mathcal{D}, & \lambda \le y \le l \end{cases}$$

Here $\mathcal{A}$, $\mathcal{C}$, $\mathcal{D}$, are constants yet to be determined and $\delta(y)$ is the delta function. The coefficient $\mathcal{B}$ is either zero (minimum overpressure) or prescribed by the limit on F' (minimum shock pressure rise). The general procedures

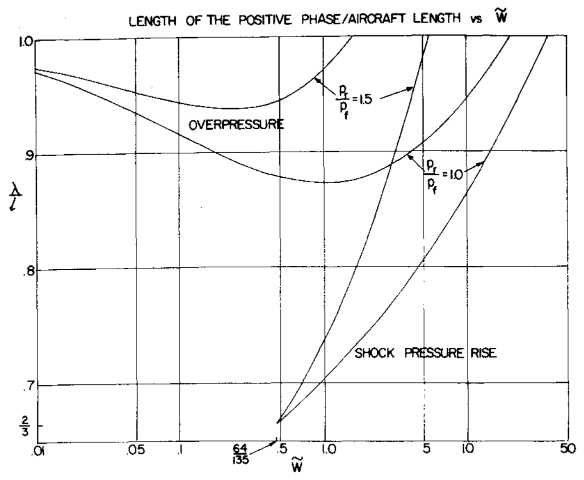

Fig. 4 Fraction of the total aircraft length used to generate the positive phase of the *F* function as a function of $\tilde{W}$ for two values of the rear-to-front shock strength ratio *R*.

outlined in Refs. 1 and 2 then lead to a system of four equations for the four unknowns $\mathcal{A}$, $\mathcal{D}$, t, and λ; $\mathcal{B}$ is given by

$$\mathcal{B} = \frac{\beta^{3/2}}{2^{1/2}\Gamma M^4}\left(\frac{2}{\pi H}\right)^{1/2}\Big/\mathrm{erf}\left(\frac{h}{2H}\right)^{1/2},$$

where $\beta = (M^2-1)^{1/2}$ and $\Gamma \equiv (\gamma + 1)/2$. The four equations that determine these four unknowns are the two area balances, the fact that the line with slope $\mathcal{B}$ starting from l must intersect the *F* function at t, and a prescription of the ratio of the rear to the front shock strength, *R*. These four equations can be reduced to two complicated algebraic equations in t and λ. These two equations depend on a single parameter related to the aircraft's weight:

$$\tilde{W} = \frac{2\Gamma}{(2\Gamma-1)}\frac{M^2}{(2\beta)^{1/2}}\left(\frac{\pi H}{2h}\right)^{1/2}\cdot \mathrm{erf}\left(\frac{h}{2H}\right)^{1/2}\left[\exp\left(\frac{h}{H}\right)\right]\left(\frac{h}{l}\right)^{1/2}\frac{W}{P_g l^2}$$

where P_g is the ambient pressure at the ground. (In Ref. 2 we used a quantity $W = \tilde{W}/4$.) When $\tilde{W}$ is less than 16/15 no front shock need occur;[18-20] when it is less than 64/135, neither a front nor a rear shock need occur.[15]

The solution of these equations must be effected numerically, but it can be done with any precision desired. The results for two ratios of the rear to front shock strength are shown in Fig. 4. One set of curves applies to minimizing the overpressure; the other set to minimizing the shock pressure rise. The solution is given in terms of the fraction of the length of the aircraft that must be used to minimize the front shock. When this ratio is equal to 1 then there is no penalty for asking that the rear shock strength be a certain fraction of the front shock strength. That is, the rear shock strength will be less than or equal to *R* times the front shock strength. Another way to view this is to interpret the quantity 1-λ as that fraction of the aircraft's length that must be used to achieve the same overpressure or shock pressure rise in the rear shock that would obtain for the front shock for an aircraft of length λ. When only the front shock is considered then the analysis can be carried through to completion analytically. Once λ and t are known the *F* function is determined, and with the *F* function determined the area development is known:

$$A(x) = 4\,\mathcal{A}x^{1/2} + \frac{16}{15}\,\mathcal{B}x^{5/2} + \frac{8}{3}\,\mathcal{C}x^{3/2} - \mathbf{1}(x-\lambda)\frac{8}{3}(\mathcal{C} + \mathcal{D})(x-\lambda)^{3/2}$$

where $\mathbf{1}(x - \lambda)$ is the unit step function. Both the Concorde and the proposed Boeing 2707 have values of $\tilde{W}$ near 2.0.

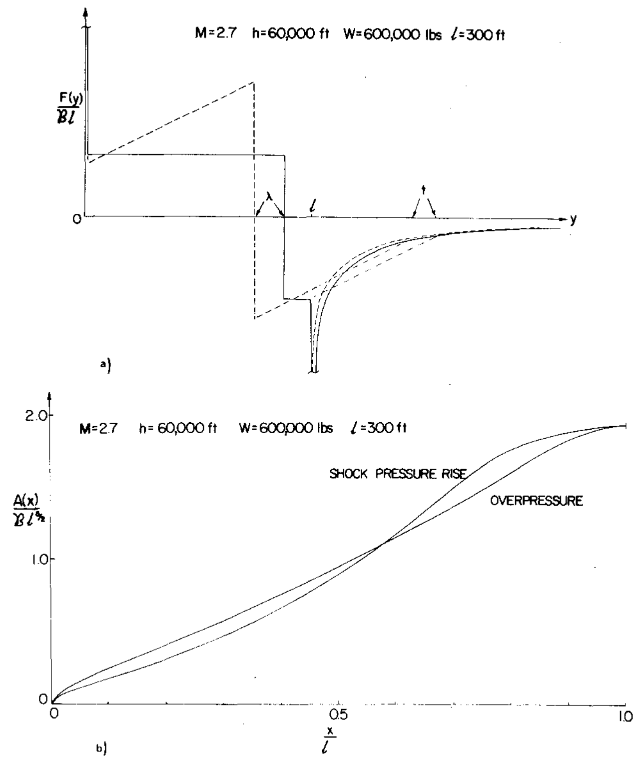

Fig. 5 *F* functions and related area developments for the nominal conditions listed that minimize the overpressure and the shock pressure rise.

Aerodynamic Results

Once the *F* function has been determined we can calculate the minimum overpressure, shock pressure rise, or impulse obtained. In the case of minimizing the shock pressure rise, there is an increase of the pressure behind the shock to a value we have indicated by p_{max}. Figures 5a and 5b depict the *F* functions and the related area developments required to minimize the shock pressure rise and the overpressure for conditions typical of U.S. SST designs. The introduction of the delta function into the *F* function violates the small perturbation assumptions upon which the aerodynamic theory underlying our calculations is based. We note, however, that this is a local failure and corresponds to a very slight nose blunting (see Fig. 5b) of the equivalent body of revolution; this is analogous to a wing with a straight leading edge normal to the flow. Wind-tunnel tests, including those of the minimum impulse shape,[26] have shown this local failure to be of no consequence. This nose blunting is so slight that it seems to present no insurmountable difficulties for the designer. Studies by Carlson et al.,[27] Kane,[28] and Ferri[29,30] indicate that these minimums may be obtained with reasonable aircraft configurations. However, to minimize the sonic boom off track, this blunting must be continued with azimuth angle for the equivalent bodies of revolution that represent the aircraft in these planes and entails a wave drag penalty. Because of the tropospheric sound speed gradient this shaping can stop at some angle short of 90°, depending upon the aircraft's Mach number and altitude. We have yet to determine the appropriate azimuthal variation of the equivalent body of revolution required to minimize the sonic boom across its corridor for a nonisothermal atmosphere. All the results we report in this section are for an atmospheric scale height of 25,000 ft; in calculating the impulse we have taken the sound speed to be 1000 fps.

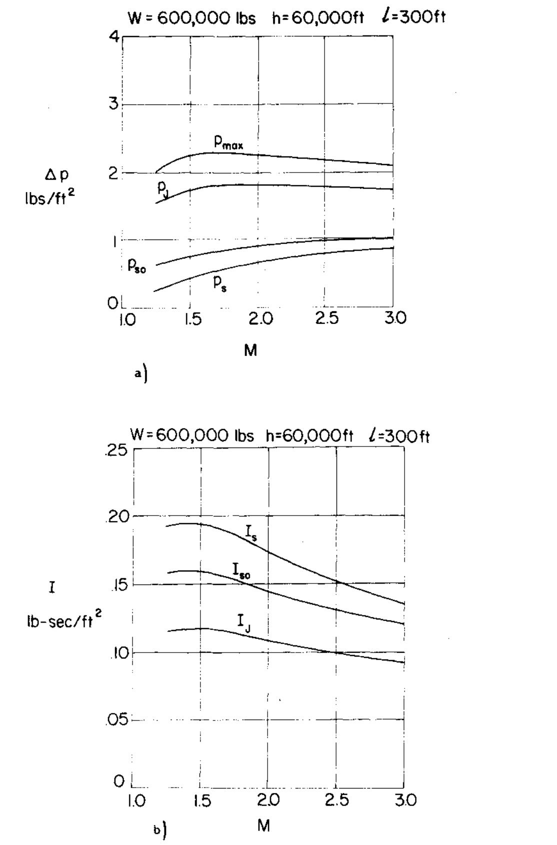

Fig. 6 Overpressures and impulses as a function of Mach number for equal strength front and rear shocks (H = 25,000 ft).

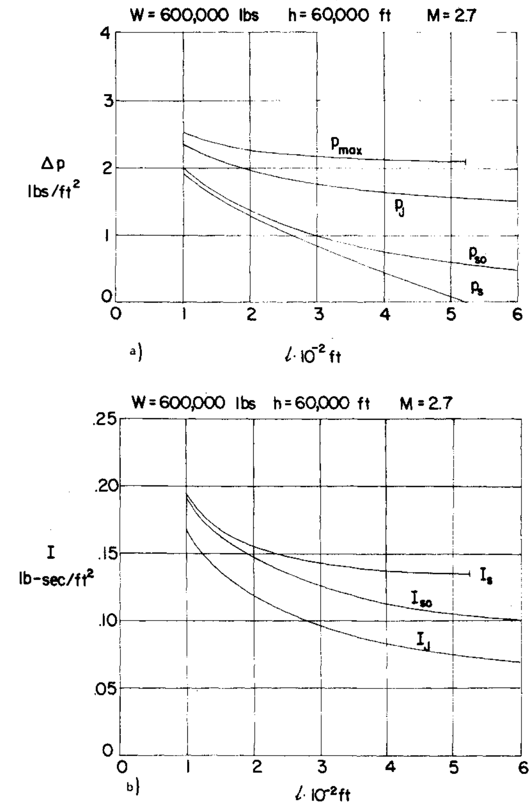

Fig. 8 Overpressures and impulses as a function of aircraft length for equal strength front and rear shocks (H = 25,000 ft).

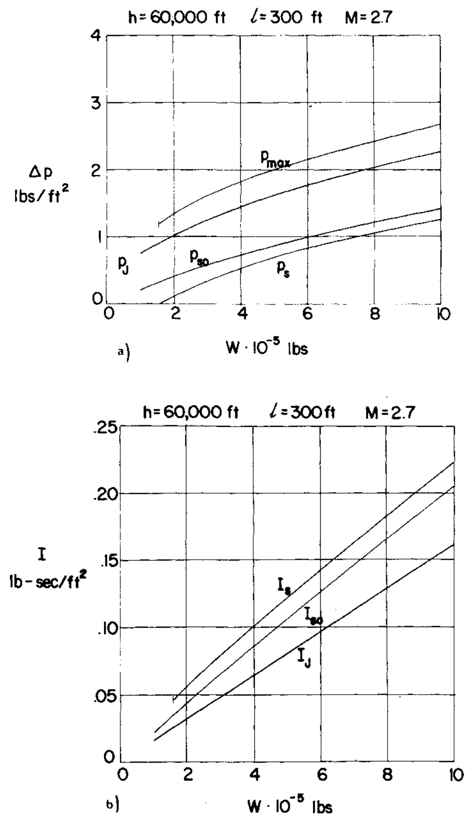

Fig. 7 Overpressures and impulses as a function of aircraft weight for equal strength front and rear shocks (H = 25,000 ft).

Figures 6a and 6b depict a variation of the overpressure and impulse (after ground reflection) for $R = 1$ as a function of Mach number for the three cases: 1) minimum impulse; 2) minimum overpressure; 3) minimum shock pressure rise. For the last case, it is also necessary to plot the maximum pressure in the signal. Observe that the overpressures are insensitive to Mach number and that the impulses decrease slightly with increasing Mach number. The variation of the overpressure and impulse with aircraft weight is shown in Figs. 7a and 7b. Note that there is a weight below which, for the prescribed conditions, there need be no shock. Also note that the impulse is essentially proportional to the aircraft weight. Figures 8a and 8b show how the overpressure and impulse vary with aircraft length; we observe that it is always beneficial to increase the aircraft's length and stretch out the pressure signal. Again, for the conditions specified, when the aircraft is 527 ft long it is no longer necessary to have a shock wave provided one is willing to tolerate overpressures of slightly more than 2 lb/ft^2. Finally, Figs. 9a and 9b show the variation of overpressure and impulse with altitude. Note that the minimum overpressure is insensitive to altitude and that the impulse for all three cases grows (exponentially) with altitude. For the lift-dominated case where the pressure perturbation is invariant with altitude, the relative strength of the perturbation to the ambient pressure grows with the altitude of the aircraft's flight. This increases the extent to which the signal will advance and this, in turn, leads to an increase in the impulse with altitude.

It should also be noted that aircraft no longer than the Concorde or the Boeing 2707 can be designed to have shock-free signatures at the ground. The length required to eliminate both shock waves and its variation with altitude weight and Mach number is depicted in Figs. 12 and 13 of Ref. 1. Thus, for example, a Mach 2.7, 600,000 lb, 300 ft aircraft can have a shock-free signature at altitudes below 30,000 ft.

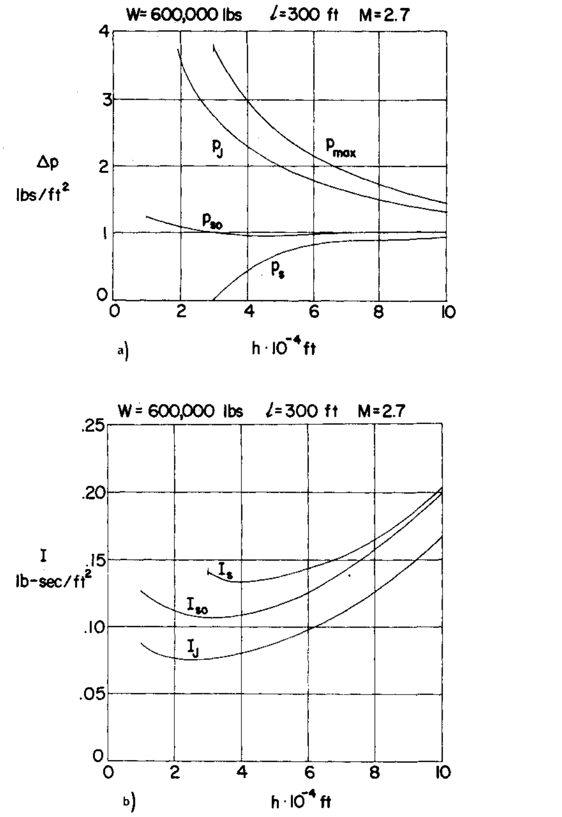

Fig. 9 Overpressures and impulses as a function of flight altitude for equal strength front and rear shocks (H = 25,000 ft).

From these results we observe that if both the overpressure and impulse are important, as we suspect they are, then we will probably need to design aircraft that fly at altitudes comparable to those used by the present subsonic jets. Surely there is no refuge in altitude for supersonic aircraft unless one is far from an optimum design, or unless we are considering hypersonic vehicles.

Computer Program for Aerodynamic Minimization

These procedures can easily be generalized to include a volume constraint because the same general form of the Whitham F function obtains. If only the lift is prescribed, then aircraft volumes up to a certain limiting value can be obtained without recourse to a Busemann biplane or ring wing. If we insist on an aircraft volume that exceeds this value and do not allow ourselves the luxury of a ring wing or similar device to achieve this volume, then there is an additional sonic boom penalty. Also, it is a simple matter to allow for the time τ the pressure takes to rise to its maximum value following the front (or preceding the rear) shock.

With the values of λ and t obtained from the four equations mentioned above, the volume at the aircraft corresponding to a given lift and center of pressure may be computed. We call this volume the free volume. If this volume is less than the prescribed volume, the equivalent body shape with the minimum shock pressure rise or overpressure will be that with sufficient lift to achieve the prescribed volume "free." This requires the solution of a modified set of four algebraic equations. J.L. Lung and W.-T. Chui[31] have developed a simple Fortran program that determines the F function and the corresponding area development required to achieve the minimum overpressure or shock pressure rise for given flight conditions, aircraft weight, volume, center of pressure, and length with a prescribed rear-to-front shock strength ratio and rise time. The input and output for this program are tabulated in Table 1.

Table 1

Input	Output
Mach number	Area development and F function for minimum overpressure
Aircraft weight, length volume and center of pressure	Corresponding overpressure and impulse
Ratio of front-to-rear shock strength	Area development and F function for minimum shock pressure rise
Rise time	Corresponding shock pressure rise, maximum pressure and impulse
Atmospheric scale height	Minimum impulse and corresponding shock pressure rise

IV. Aircraft Operations to Reduce Sonic Boom

Having found no relief from the sonic boom by operating supersonic aircraft at higher altitudes, we may turn to other alternatives. These include operating the aircraft at a low enough Mach number that the acoustic signal it emits is refracted by the atmosphere before it reaches the ground, high altitude-high speed flight, or unsteady operation. The use of maneuvers by supersonic aircraft to reduce the sonic boom in a localized region have been discussed by Batdorf[32] and Ferri.[29] While such procedures are feasible they seem impractical, as the sonic boom must be reduced over much or all of the flight path crossing land areas.

Transonic Operation

One way to avoid having the sonic boom reach the ground is for an aircraft to fly slow enough that its Mach number, based upon the speed of sound at the ground, is less than one. For the standard tropospheric temperature gradient this corresponds to a stratospheric flight Mach number of about 1.15. The operation of the aircraft at speeds below this so-called "threshold" Mach number has the disadvantage of a lift-to-drag penalty but the advantage of increased productivity compared to subsonic transports. Under such operations, the signal generated by the aircraft is refracted by the sound speed gradient, and the shock wave terminates in a reflected wave above the ground. The detailed structure of the waves where they terminate near the sonic line are complex and not fully understood.[33-36] However, the details of the sound field below the caustic surface where refraction occurs can be determined adequately from linear theory. K.-Y. Fung[37] has carried out this computation and finds that below a caustic

$$\frac{Cp}{Cp_N} = \frac{1}{2\pi(2)^{1/2}}\left(\frac{y_N}{|y|}\right)^{1/4} S\left(Ut/l,\ \frac{2}{3}\left(\frac{(M^{2\prime}y^3)^{1/2}}{l}\right)\right)$$

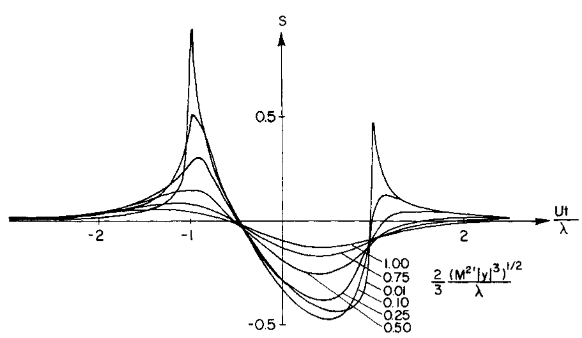

Fig. 10 The function S that describes the pressure signal below a caustic.

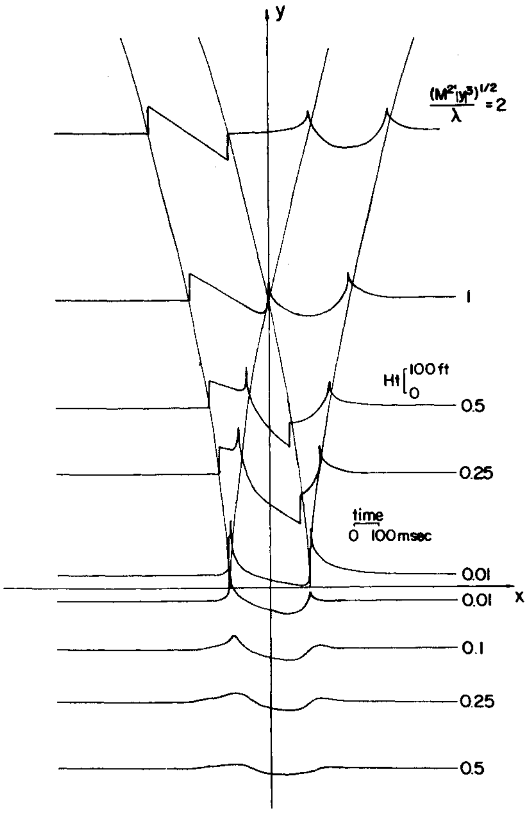

Fig. 11 Linear behavior of an incident *N*-wave near a caustic. $M^2 = 7.7 \times 10^{-6}$ ft^{-1}, $\lambda = 150$ ft, and $U = 1000$ fps.

where U is the aircraft's speed, t the time, y the vertical distance measured from the caustic, $2l$ the length of the incident N-wave, Cp_N its pressure coefficient a height y_N above the caustic surface, and $M^{2'}$ is the (constant) gradient of the square of the aircraft's Mach number based on the local speed of sound. Figure 10 depicts the variation of S with time for various values of y. These results may be summarized succinctly by noting that the distance below the caustic, y, at which the maximum pressure is comparable to the overpressure of the incident signal at the same distance above the caustic, is approximately $kl^{2/3}$, where k is approximately $1.7(\text{ft})^{1/3}$ for the standard atmosphere. For $l = 150$, the maximum pressure 40 ft below the caustic has the same magnitude as the overpressure of the incident N-wave 40 ft above the caustic. In an additional 900 ft the maximum pressure will decrease by a factor of 10. Figure 8 indicates the variation of S with time for various values of y. From the variation of S with t we can estimate the maximum frequencies in the pressure signature. For $l = 150$ ft, the characteristic frequencies are about 6 and 1 Hz at about 1900 ft and 8700 ft below the caustic. Far below the caustic, the pressure decays as $(y/l)^{-13/4}$.

The equation that describes the behavior of a weak, discontinuous pressure signal near a caustic is nonlinear and consequently difficult to solve. While the linear version of this equation can be solved by Fourier transforms,[34,37] as well as by other means, the resulting solution, while physically simple, is represented by a complicated combination of hypergeometric functions. This linear solution, which becomes algebraically singular as the signal approaches the caustic and gives rise to a reflected signal that is logarithmically singular, is depicted in Fig. 11 for conditions typifying threshold operation. Below the caustic the behavior is, of course, that given above.

Because the signal amplifies without limit as it approaches the caustic, a nonlinear description is essential. Efforts to determine this nonlinear behavior have met with some success.[36,38] From these results we can deduce that the shock waves terminate in weak compressions formed by coalescing waves emanating from a distorted "sonic" line; the sonic line itself runs into this shock. Above the junction with the sonic line the shock becomes stronger and nearly normal; behind this normal portion of the shock the flow is subsonic in a coordinate system fixed to the shock. This normal shock presumably joins the incoming and reflected wave at a single point. The variation of the jump in pressure coefficient through the reflected shock has been estimated by Gill and Seebass.[36]

Hypersonic Transport

At the present time it is not possible to predict the sonic boom of general hypersonic aircraft. The absence of a complete theory for hypersonic boom does not preclude the possibility of determining the strength of the leading shock wave as well as the impulse of hypersonic vehicles that are drag-dominated. Presumably these impulses are smaller than those associated with higher lift-drag ratio aircraft. In the drag-dominated case the flow field far from the vehicle will be that produced by energy released by the vehicle per unit time, that is, the product of the vehicle's drag with its velocity. Thus, we can invoke the hypersonic equivalence principle of Hayes to determine the flow near the body; a simple inviscid Burgers equation describes its evolution far from the body. These two flow fields can be coupled by several expedients. One is to simply recast the expansion for the shock pressure rise for finite counterpressure so that it agrees with the decay of a weak shock in a homogeneous atmosphere. This result can then be corrected for atmospheric effects.[39] Such a procedure gives reasonable results; however, it does not provide an estimate of the impulse of the signal and a more direct matching of the far-field behavior with the numerical results must be used.[40-42] Numerical calculations for the cylindrical blast wave have been performed by Plooster[43] and recently, using a more refined technique, by B. Tiegerman.[42]

If we consider a hypersonic vehicle with at most a moderate lift-to-drag ratio, then we can use these results to determine the overpressure and the impulse as a function of altitude for a given aircraft drag. (See Ref. 39 for the modifications due to flight path angle.) Using the numerical calculations of Tiegerman, and considering the atmosphere to be isothermal, we conclude that for drag-dominated vehicles the overpressure is governed by

$$\Delta p = 0.59\,\frac{P_g^{5/8} D^{3/8} \exp(-h/8H)}{H^{1/4} h^{1/2}}$$

while the impulse obeys

$$I = 0.16\,\frac{P_g^{1/4} D^{3/4} \exp(h/4H)}{h^{1/2} a_g}$$

where D is the aircraft's drag, h its altitude of operation,

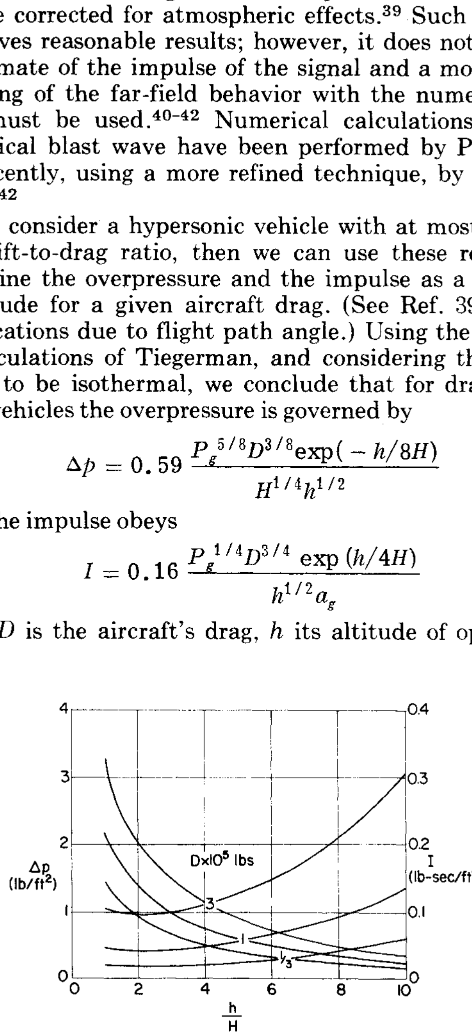

Fig. 12 Overpressure and impulse as a function of altitude for drag-dominated hypersonic aircraft ($H = 25{,}000$ ft).

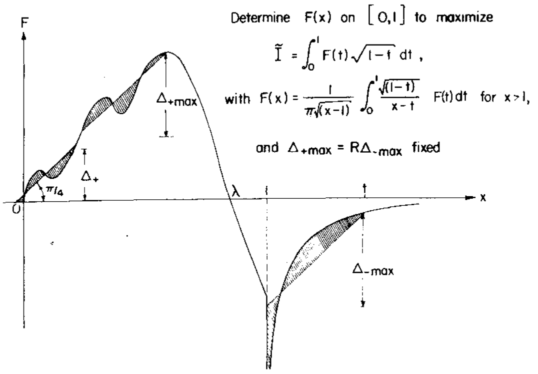

Fig. 13 Mathematical formulation of the minimization problem; the shaded areas on each side of the lines with unit slope must balance.

H the atmospheric scale height, a_g the speed of sound at the ground and P_g the ambient pressure there. These results are sketched in Fig. 12 for various aircraft drags, which may be interpreted as the weight of the aircraft divided by the lift-drag ratio. Note that the overpressures and impulses obtained at realistic flight altitudes (h/H = 5 to 6) may be less than those achieved by the minimum overpressure designs for supersonic aircraft. As a consequence of the exponential growth of the disturbance pressure relative to the ambient pressure, for a fixed drag, the impulse increases exponentially with altitude; thus, while the overpressure continues to decrease with altitude, the impulse grows markedly. Consequently increasing the flight altitude further may decrease the "acceptability" of the sonic boom of hypersonic transports. These results must not be applied at altitudes where the mean free path is no longer very small compared to the square root of the vehicle's cross-sectional area.

These results are in accord with those reported by Pan and Sotomayer[40,41] for a homogeneous atmosphere and later modified by them.[44] To the extent we are able to compare these results with hypersonic booms measured during Apollo 15 and 16 re-entry,[45,46] we find reasonable agreement. With certain modifications they can be used to predict the sonic boom generated by rocket exhaust plumes[47-48] at high altitudes. Of course the plume diameter must be many mean free paths for the modified results to apply, but this is usually the case.[49] The ultimate viscous decay of sonic booms that have become very weak is discussed by Sachdev and Seebass.[50]

IV. Conclusions

By resorting to careful aerodynamic design to minimize the sonic boom overpressure, it seems that aircraft can be designed that would achieve overpressure levels just below 1 lb/ft^2 (for both the positive and negative phases of the pressure signature) and impulses of about 1/10 lb/sec/ft^2. These numbers are not too different from the sonic boom generated by the SR-71,[51] and experience with SR-71 overflights should give some indication of whether or not overpressures and impulses of this magnitude will prove acceptable. If they do not, then there is refuge in the operation of the aircraft at slightly supersonic speeds. Whether increased utilization can pay for the increased operating costs in such flights is unclear and there is always the attendant danger that the caustic lying between the aircraft and the ground will inadvertantly intersect the ground. Unfortunately, there does not seem to be much refuge in high-altitude hypersonic flight either, indicating that hypersonic transports might also be constrained to supersonic operation over water. Some current aerodynamic effort is directed toward increasing the efficiency of SST generation aircraft in order to make them economically attractive even though limited to subsonic operation over land.[52-53] Since the reductions which can be achieved by aerodynamic minimization could bring SST's sonic boom into the range of overpressures and impulses that may prove acceptable, we must further delineate sonic-boom "acceptability" and determine whether or not the idealized minimums discussed here can be approached in practical aircraft designs.

Appendix

The problem of minimizing the sonic boom can be reduced to that of maximizing the aircraft's lift with the shock pressure rise $\Delta\pm$, constrained. For example, to minimize the shock pressure rise, after proper account is taken of the evolution of the waveform, the problem can be reduced to that posed in Fig. 13. The strength of the front and rear shocks is determined by balancing areas of an appropriately scaled F function with lines of unit slope. The problem reduces to one of maximizing the lift, which is proportional to $\tilde{I}$ with the maximum front and rear shock strengths specified. One can show that the proper solution is one where the scaled F begins with a delta function and then increases linearly with a nondimensional coordinate x for x between 0 and a switching point λ; the scaled F must then jump to some negative value and again increase linearly with x until $x = 1$.

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